



# D-GEO FLOW

Groundwater flow and  
piping calculation module



Installation manual | Tutorial | Scientific background





## D-GEO Suite D-Geo Flow

Installation manual, tutorial and scientific manual



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Installation manual, tutorial and scientific manual

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# D-GEO Suite D-Geo Flow

Installation manual, tutorial and scientific manual

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# Summary

D-Geo Flow is free to use groundwater flow and backward erosion piping software for soft soil engineering.

This document contains the installation manual, tutorial and scientific manual of D-Geo Flow

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# 1 Introduction

## 1.1 D-Geo Flow

D-Geo Flow is software for groundwater flow and backward erosion piping analysis.

Deltares dares to share. D-Geo Flow is free to use.

**Note:** In its current version, the software can only perform a groundwater flow calculation for the sake of a Backward Erosion Piping (BEP) analysis. This means, for example, that it only contains open and closed boundaries and only supports steady state analysis. Groundwater flow capabilities to, for example, determine a phreatic line, will be added to the software later on.



## 1.2 Deltares

Deltares is an independent institute for applied research in the field of water, subsurface and infrastructure. Throughout the world, we work on smart solutions, innovations and applications for people, environment and society. Knowledge is our core business. Deltares employs over 800 people and is based in Delft and Utrecht, the Netherlands.

## 1.3 Rijkswaterstaat

Rijkswaterstaat is the executive agency of the Ministry of Infrastructure and Water Management, dedicated to promote safety, mobility and the quality of life in the Netherlands.

## 1.4 Contents of this document

This is the user manual for D-Geo Flow. The user manual contains the following sections:

- Installation Manual;
- Tutorial;
- Scientific background information.

# Installation manual



## 2 Installation manual

### 2.1 Downloading D-Geo Flow

D-Geo Flow is free to use and available both through the IPLO and the Deltares website. To download D-Geo Flow from the Deltares website, go to [the Deltares website \(www.Deltares.nl\)](http://www.Deltares.nl) and click on Software to find it amongst our products.

After downloading the software, you can either choose to run the software directly from the browser or double click on the downloaded .msi file from Windows Explorer. The software does not require any privileges to install, besides the fact that the user needs to be allowed to run a .msi file. If you don't have the rights to install software, please contact your system administrator.

**Note:** When you start the installation, please note that the installation is from a trusted publisher: Deltares.



The installation wizard starts and will guide you through the installation process.

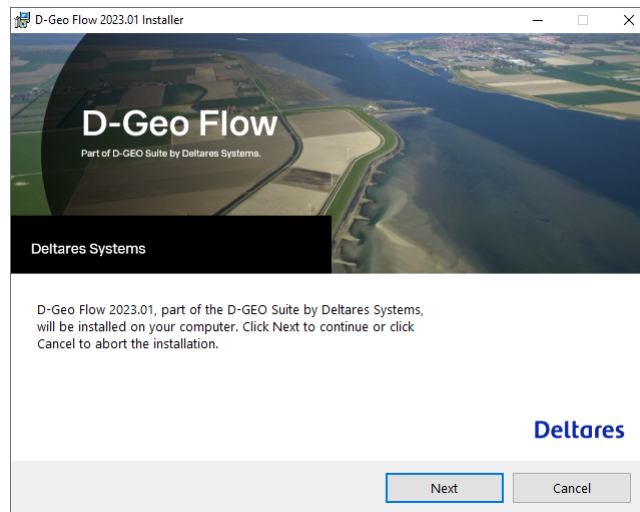


Figure 2.1 *Opening screen installation wizard*

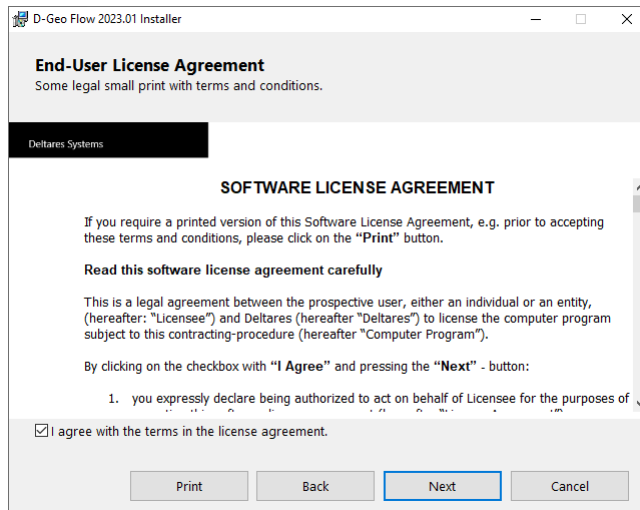


Figure 2.2 Agree with license

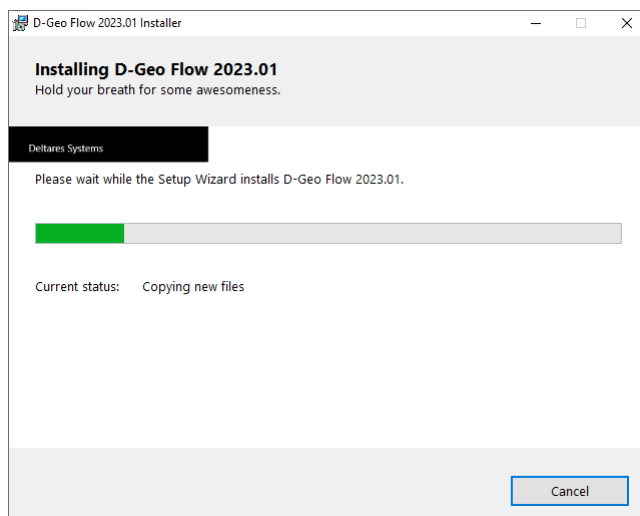


Figure 2.3 Installing the software

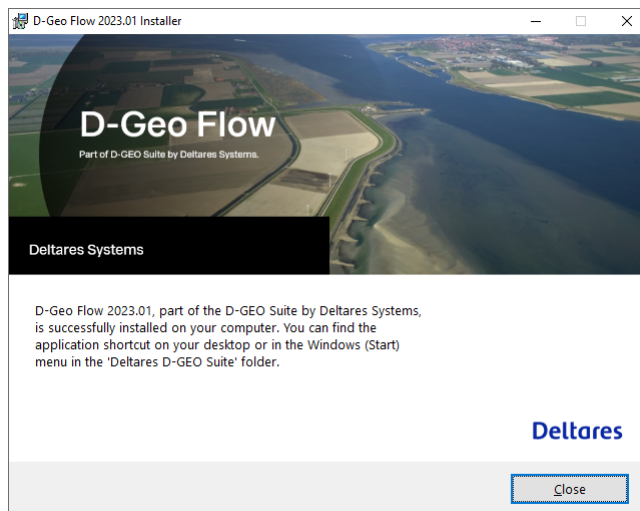


Figure 2.4 Finishing the installation

## 2.2 Verifying the installation

After installing D-Geo Flow, you can start it from your start menu where it initially appears at the top, or find it using the search function in your taskbar.

To check whether the installation was successful, you can open the installed .flox file that accompanies the installation called "Tutorial.flox". By doing so, the following geometry will appear.

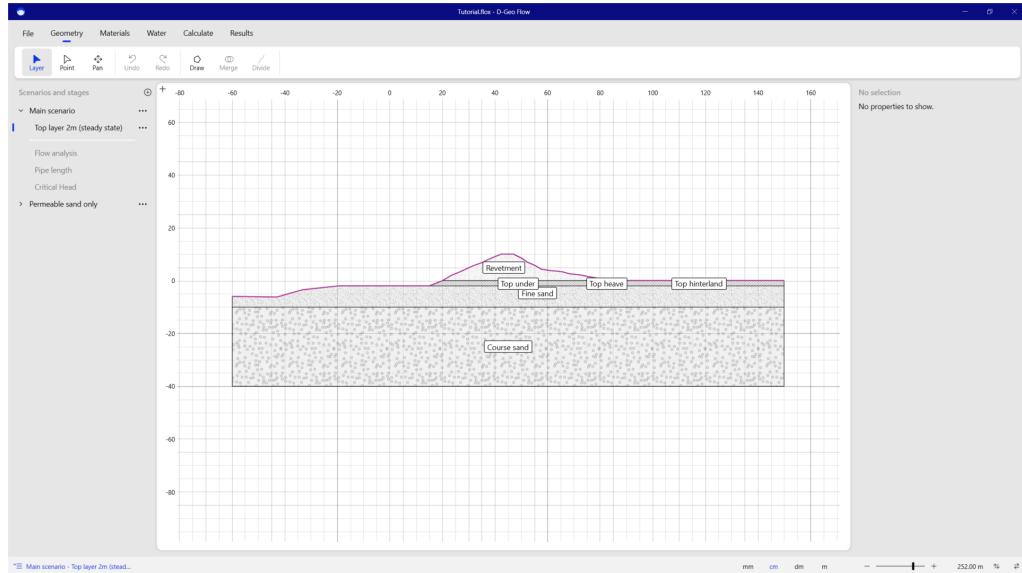


Figure 2.5 The tutorial file

Next, go to the Calculate tab and press the "Calculate" button in the ribbon. The software should start calculating, and after a while, the calculation result should appear.

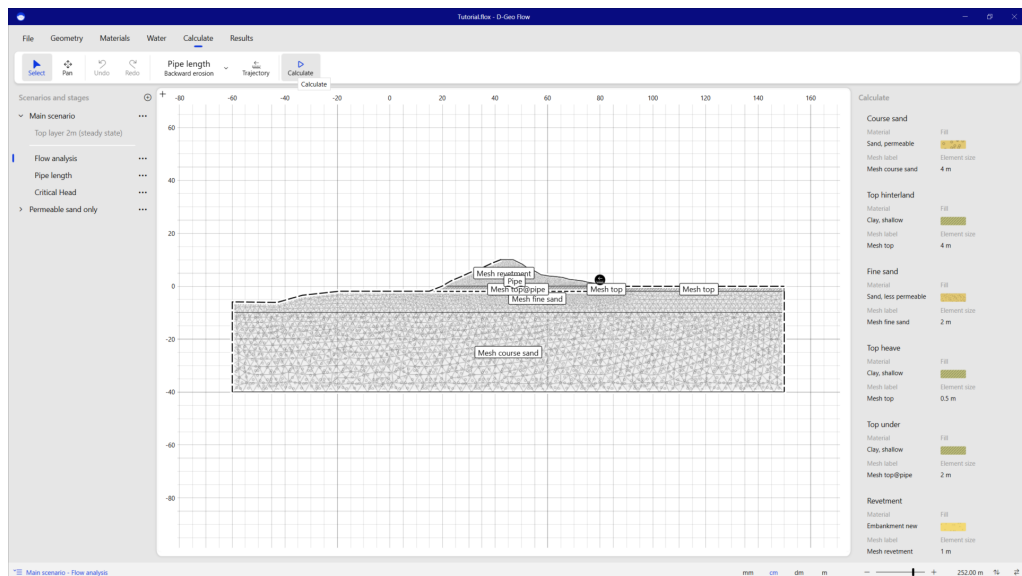


Figure 2.6 Calculate the tutorial file



If you see the "Calculating" window pop up, the software runs and the installation is performed correctly.

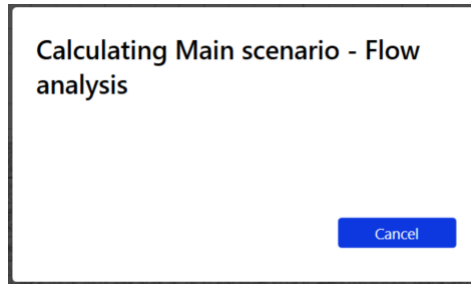


Figure 2.7 *The calculation runs*

# Tutorial

# 3 Tutorial: Critical head analysis

This tutorial will guide a user through most of the application by walking through the steps to determine the ground water flow, the pipe length and the critical head of an embankment. The tutorial gives thereby an overview of D-Geo Flow. Some detailed aspects may not be addressed. The software ought to be sufficiently intuitive to discover the use without having each field explained.

**Note:** This example is for instruction purposes only. It is not a guide to perform a complete calculation.



## 3.1 Creating a new file: Project description

When opening D-Geo Flow, it opens at the opening screen. Here the user can open an earlier used project, or start a new project. At the right top, the user can give feedback to Deltares regarding the software, and the user is informed when to install a new version when the current version is outdated.



Figure 3.1 Backdrop

For the sake of this tutorial, a new project is started from scratch by pressing the "New" button. An empty canvas is shown.



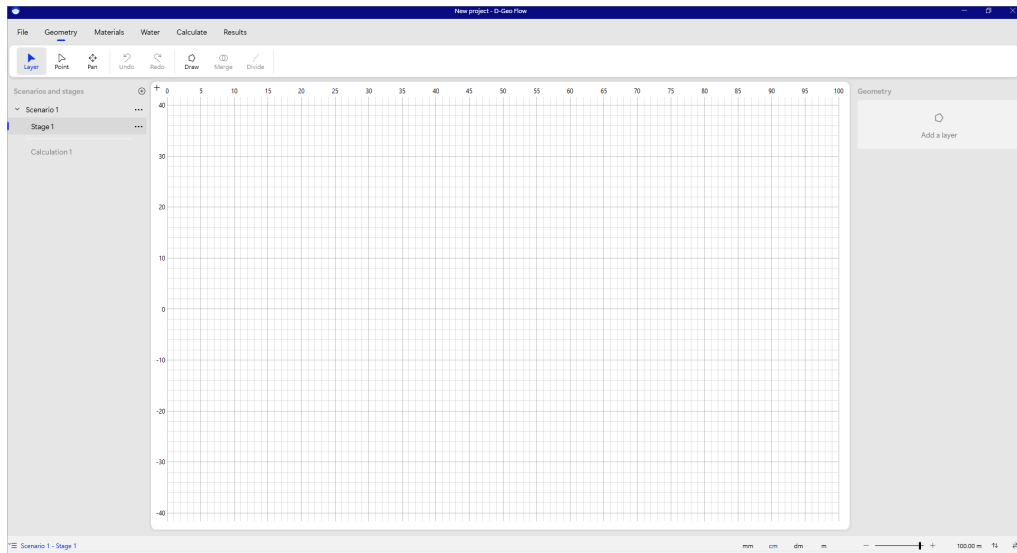


Figure 3.2 *New canvas*

By going to the "File" the user will see additional file options on the left, and an overview of recent projects on the right.

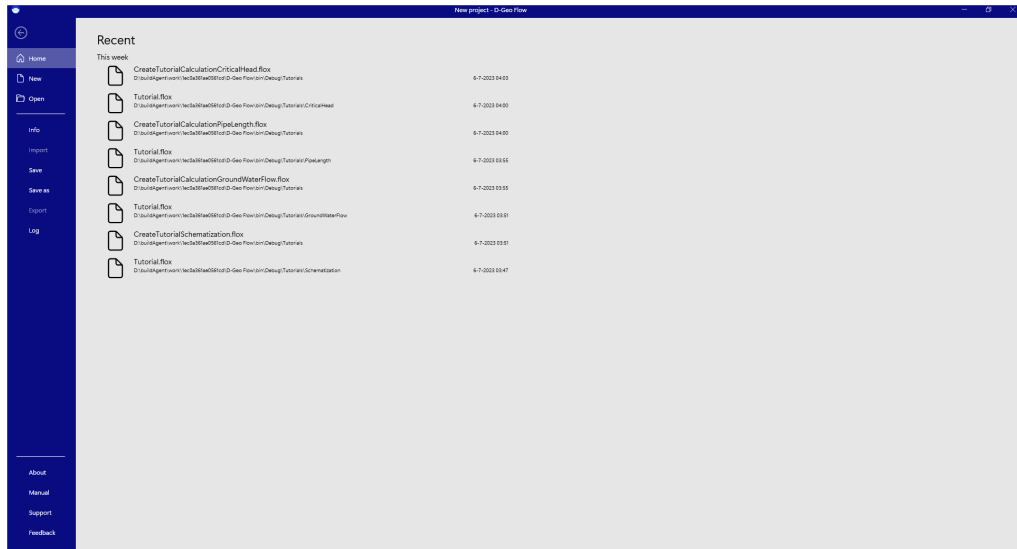


Figure 3.3 *Home screen*

By selecting "Info", the user will see the menu to enter project information to describe the current project.

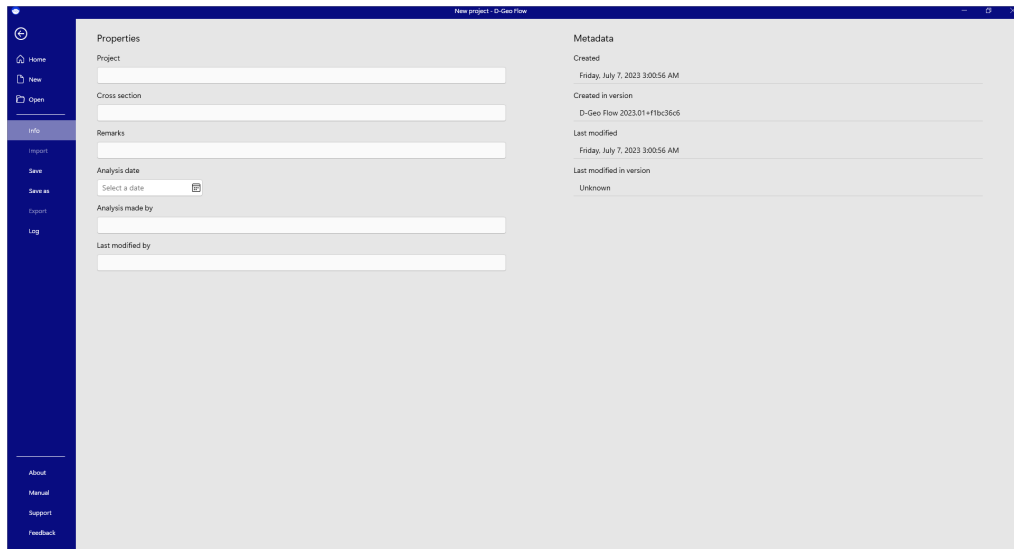


Figure 3.4 *Entering the project info*

**Note:** D-Geo Flow remembers the version the project is started in, and the version the project is last saved in. If the schematization is fully made in the Graphical User Interface (GUI) the checksum will always be valid. If it has been altered outside the GUI, the GUI cannot guarantee its consistency and the checksum will be invalid. It does not mean that the input is invalid, only that the GUI cannot completely guarantee its consistency. The user is advised to be more cautious when reviewing a result with an invalid checksum.

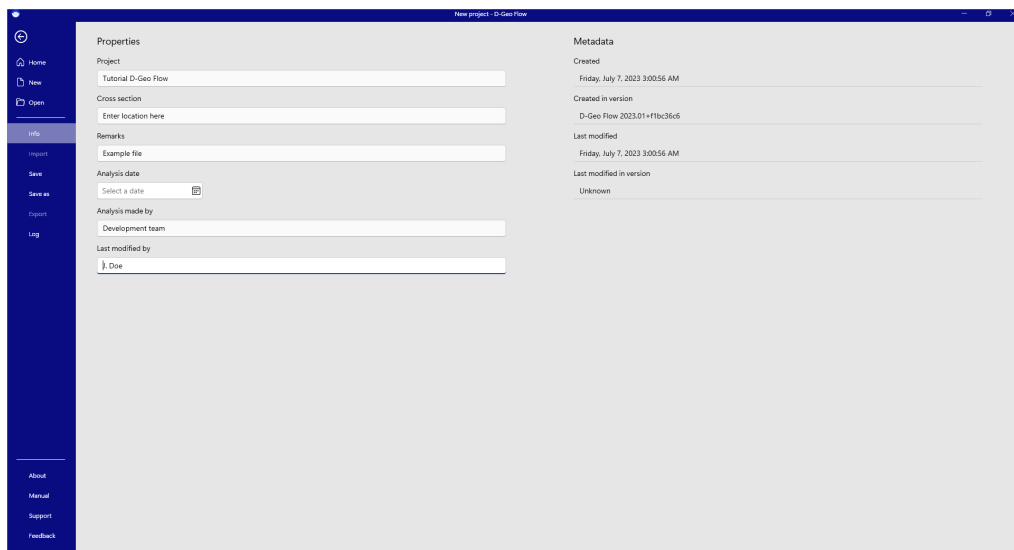


Figure 3.5 *Project info full*

The "About" section shows the contributors to the software. Kratos Multi-Physics contains the FEM engine for the analyses, the software is financially made possible by BOI, HWBP and Deltares.

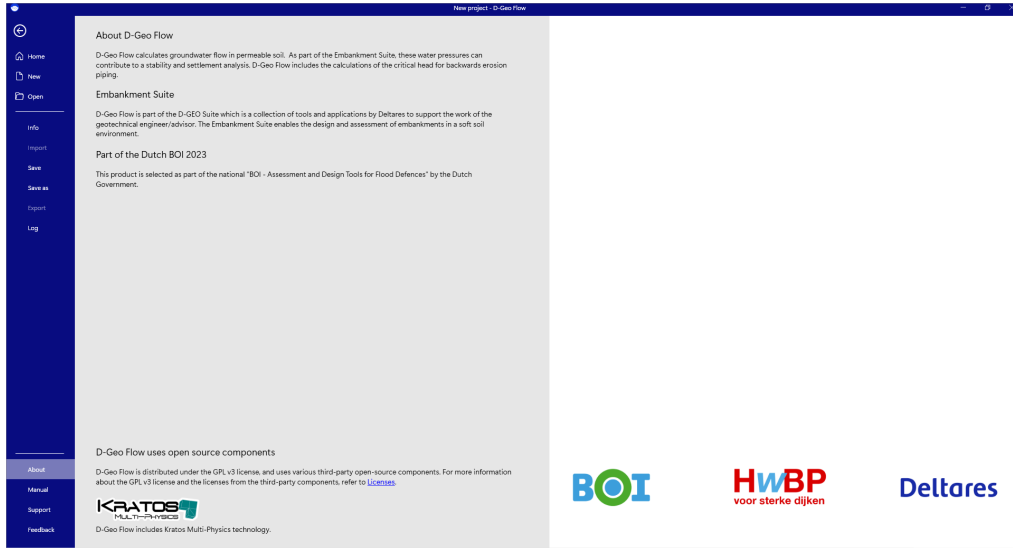


Figure 3.6 *About*

The level of support is dependent upon the support level of the client. Without an additional support contract, the user can only get support through the Deltares website or by contacting us via email.

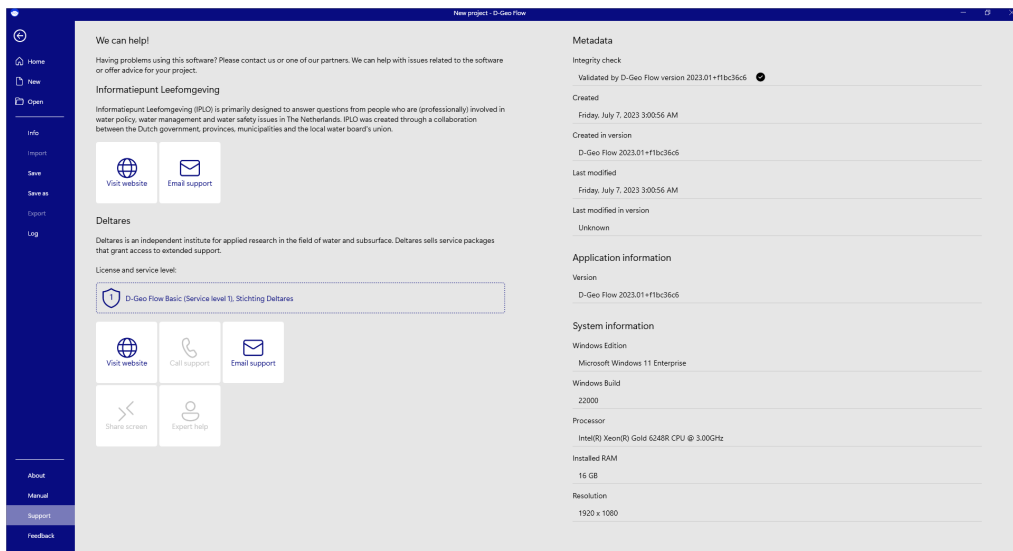


Figure 3.7 *Support*

## 3.2 Start a Schematization

Strictly speaking, it is not necessary to draw the embankment correctly for a Backwards Erosion Piping analysis as the shape of the embankment is not relevant for the mechanism. In this tutorial, we will do so anyway as it makes the results of analysis look better. Drawing of the embankment and the subsoil can be performed with the "Draw" button under "Geometry."

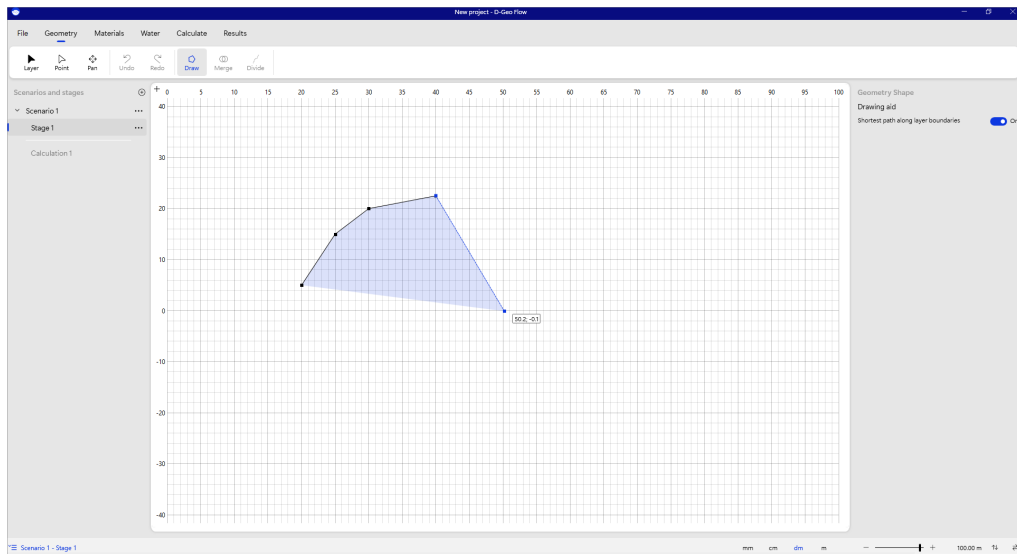


Figure 3.8 Draw shape

After the shape is drawn, its label can be edited for proper display in later sections.

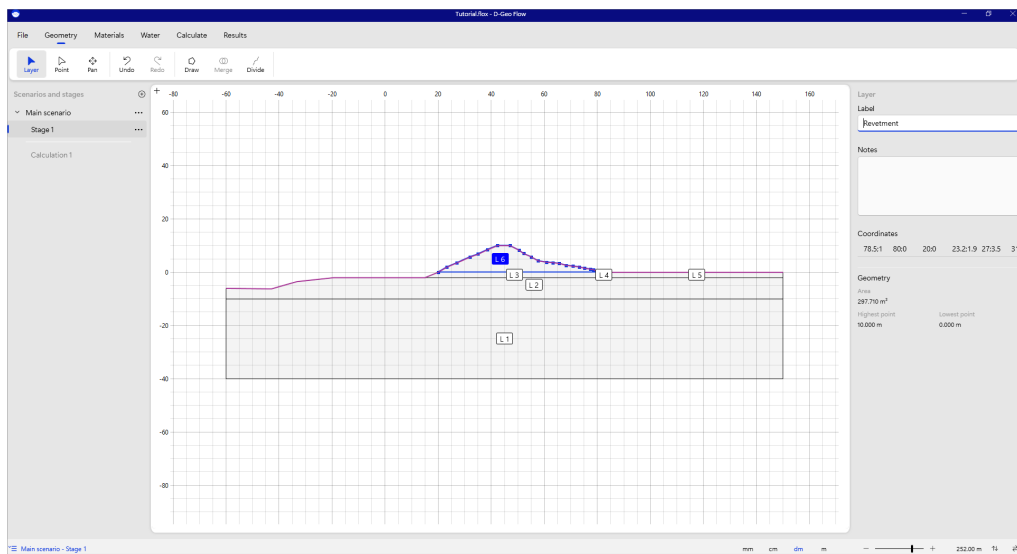


Figure 3.9 Label shape and scenario

The name of the scenario and the calculation step in the cross section can be entered in the label as well.

**Note:** Currently, only steady state analyses can be run. Therefore, only one time step



can be defined per scenario. This can be different in other applications in the Embankment Suite.

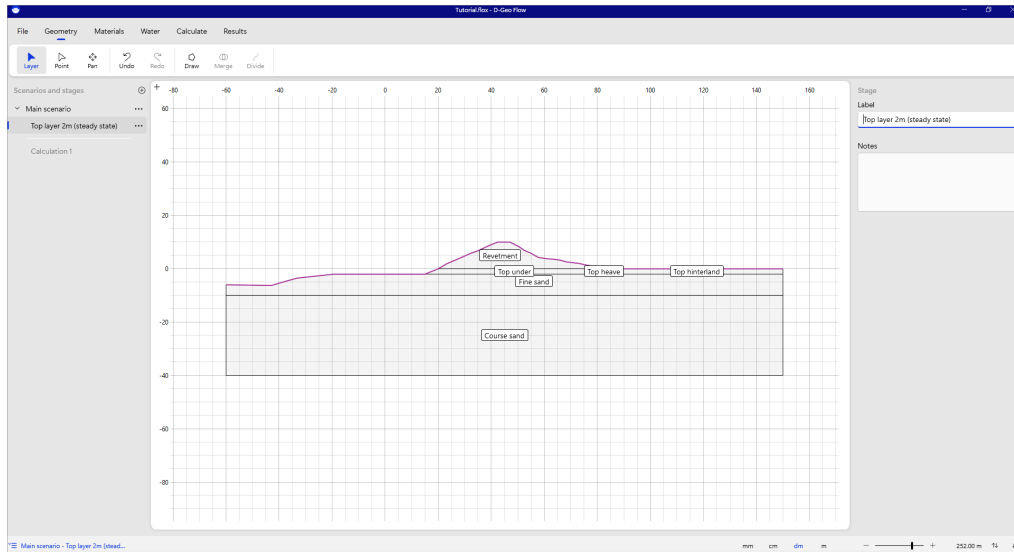


Figure 3.10 Label the name of the cross section

If the drawn points are not correct, they can be moved by selecting and dragging them with the mouse. Points can also be edited by selecting them and updating the properties in the inspector.

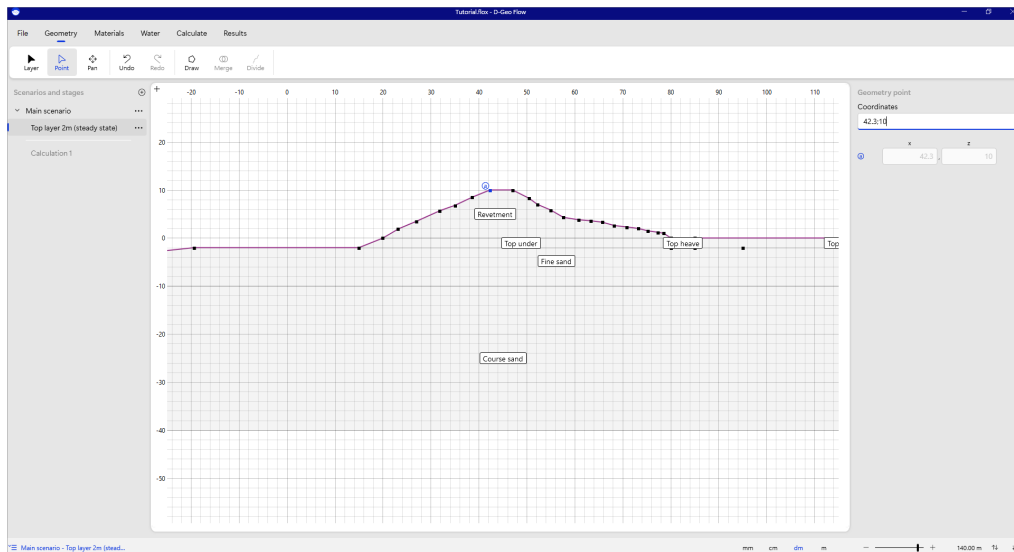


Figure 3.11 Move point

**Note:** Zooming can be done by scrolling the mouse wheel. On the canvas, this will zoom the whole geometry. On a single axis, this will only zoom the specific axis.



**Note:** You can pan by pressing and holding the scroll wheel of the mouse.



**Note:** Mouse clicks can be rounded off to the millimeter, centimeter, decimeter, or meter, depending on the selection of units at the bottom right of the screen.



After the geometry is drawn, materials can be assigned in the "Material" tab by selecting a layer and opening the "Assign material" menu. The user can choose from a set of default materials or define add custom material using the "Add material" button in the "Assign material" menu.

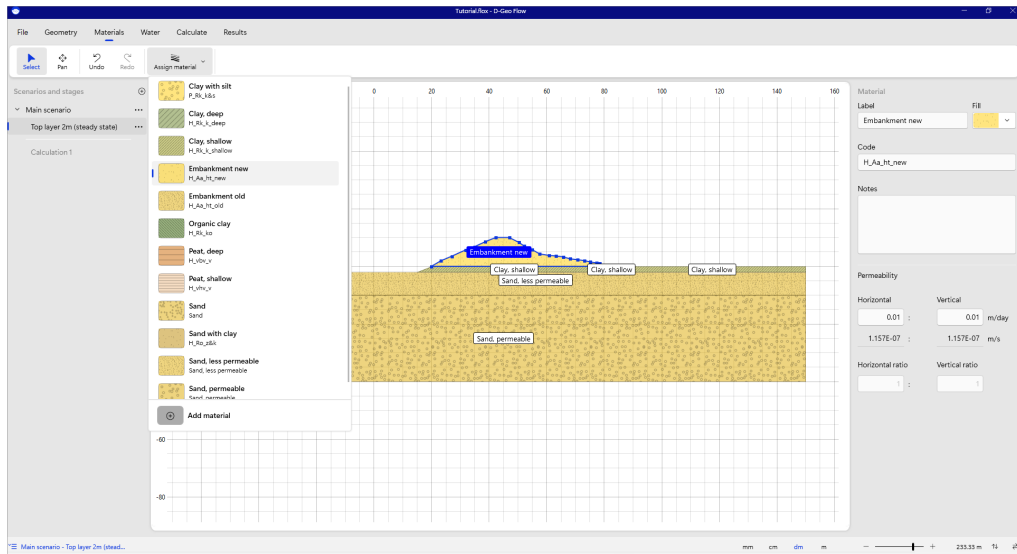


Figure 3.12 Assign material

For all materials, the user can choose a custom color and shading.

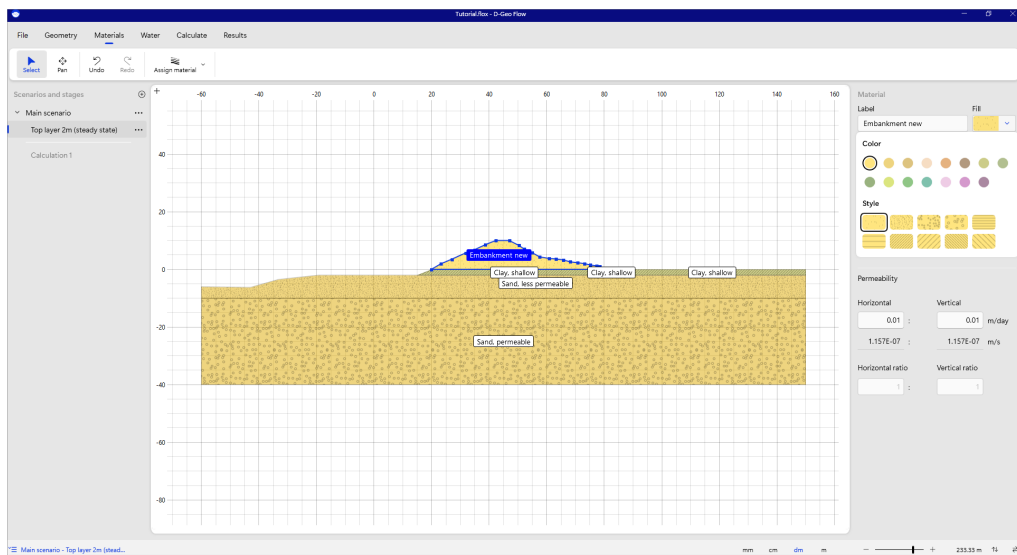


Figure 3.13 Color and shading of material

Material properties can be changed in the inspector. Note that a "notes" field is available for a possible audit trail of the decisions made in the project.



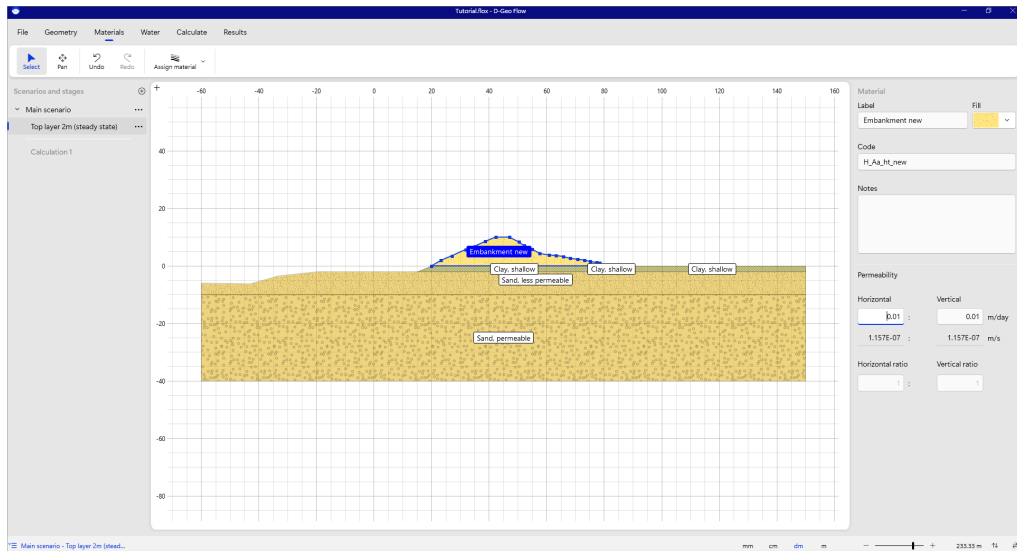


Figure 3.14 *Change permeability*

In the "Water" tab, the user can define the boundary conditions for the groundwater flow analysis. Initially, all the edge boundaries are closed. All boundaries between materials are open. By pressing the "Boundary" button, the user can replace a closed boundary with a fixed head boundary.

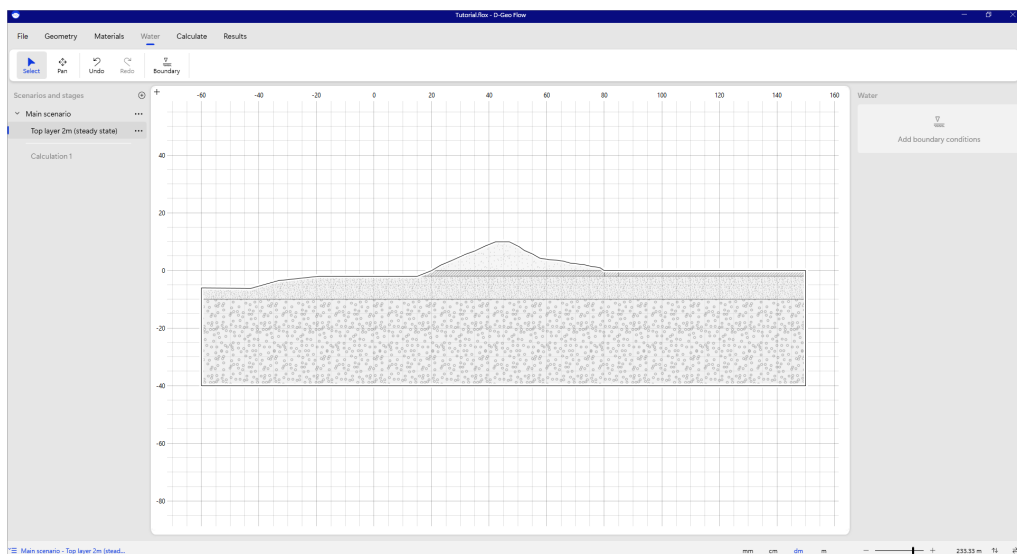


Figure 3.15 *Boundary conditions - all closed*

The label can be changed, and a value of the fixed head can be entered in the inspector.

**Note:** Currently, boundaries can only start and finish at existing geometry points.



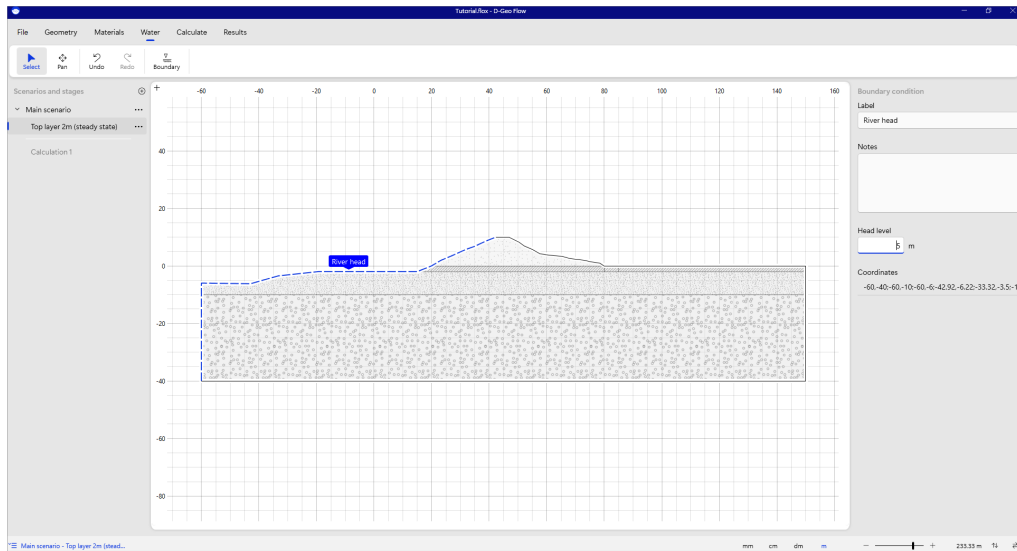




Figure 3.16 *Fixed head boundary*

All fixed head boundaries will be used for the calculations in the active scenario. If different heads need to be analyzed, the user needs to define a new scenario to do so. This can be done by adding a new scenario using the "+" button next to "Scenarios and stages", or by duplicating a scenario.

**Note:** Boundaries are closed and therefore the entire geometry will be saturated. No phreatic line is calculated, the software allows for suction (negative water pressures) in the geometry. 

**Note:** The software does not cut off the maximum pore pressures due to uplift. If you notice that the water pressures are higher than physically possible, you need to define an uplift boundary, or, like in the example below, a heave boundary. 

In the figure below, a heave boundary is defined. It is assumed that the maximum potential will be 0,3 times the thickness of the blanket layer.

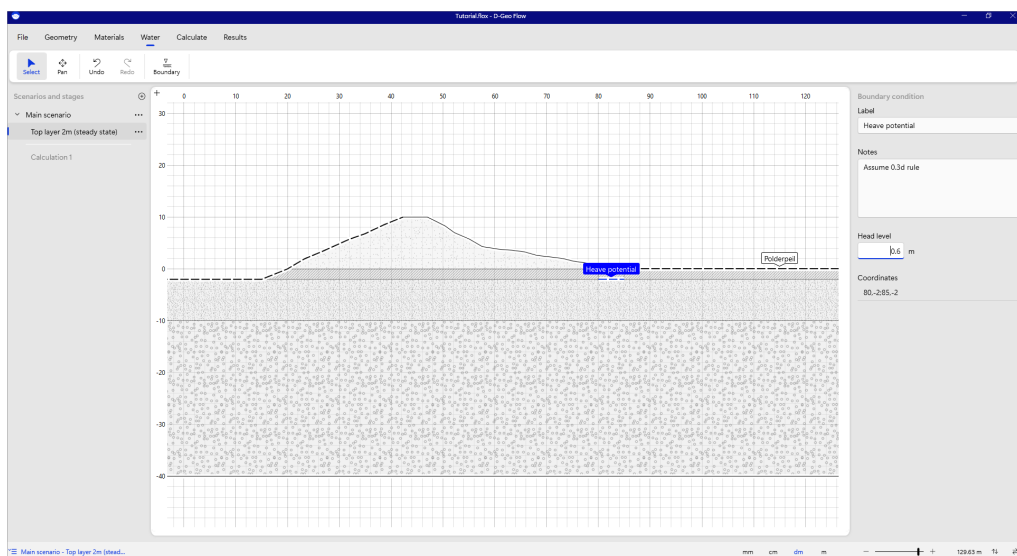


Figure 3.17 *Enter heave potential*

After all the input is prepared, the calculation can be run.

### 3.3 Groundwater flow calculation

A groundwater flow calculation can be performed by selecting "Flow" from the "Calculate" tab.

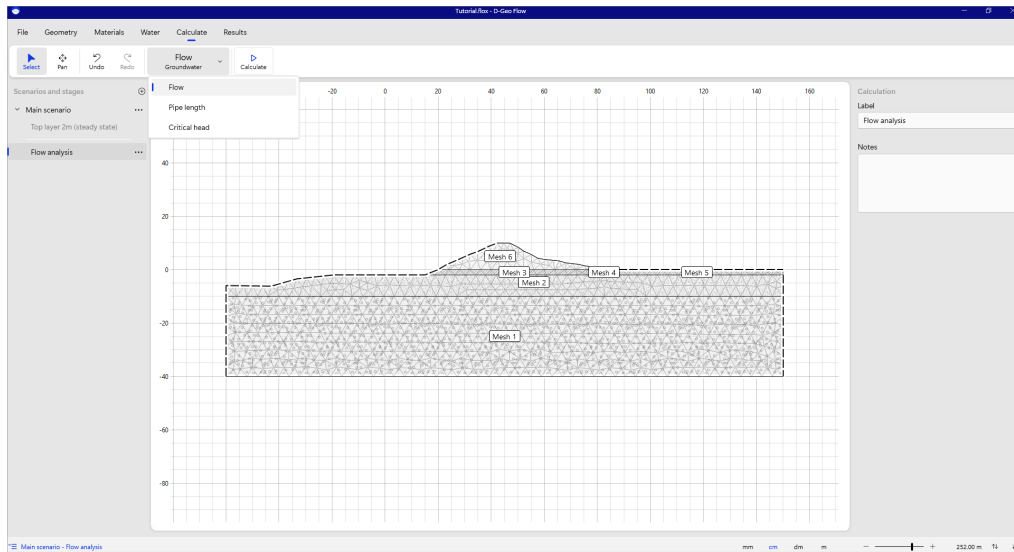


Figure 3.18 Groundwater flow calculation

The mesh is a calculation setting for the groundwater flow calculation. The number of elements determines the calculation time of the analysis. It is advised to start with a coarse mesh and later inspect whether refining the mesh influences the result. The coarseness of the mesh can be defined in the inspector. Note that if a different coarseness is defined for different layers, or for a layer and the pipe, the finest boundary will be adhered to.

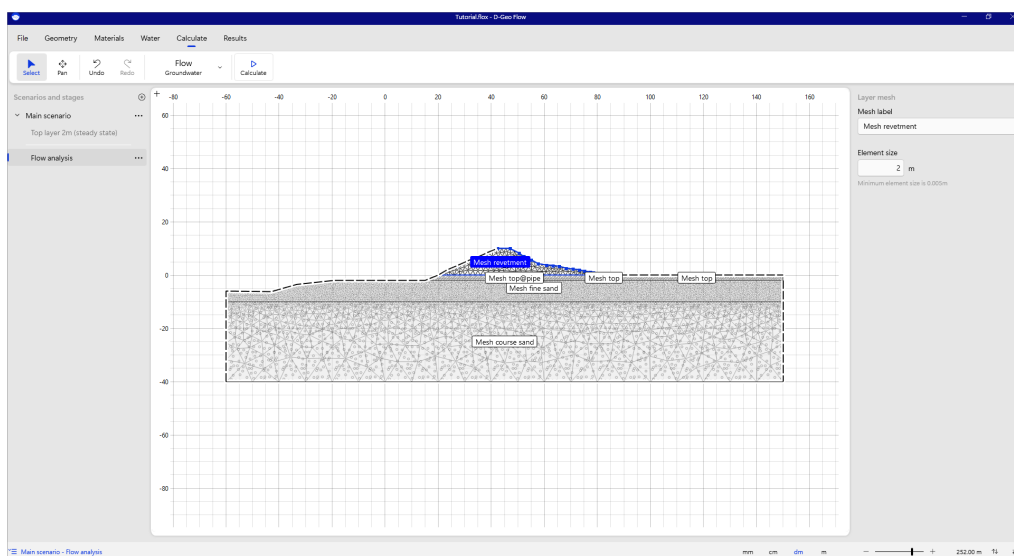


Figure 3.19 Mesh resolution

The pictures below show a coarse mesh of the aquifer. This is a good initial choice.

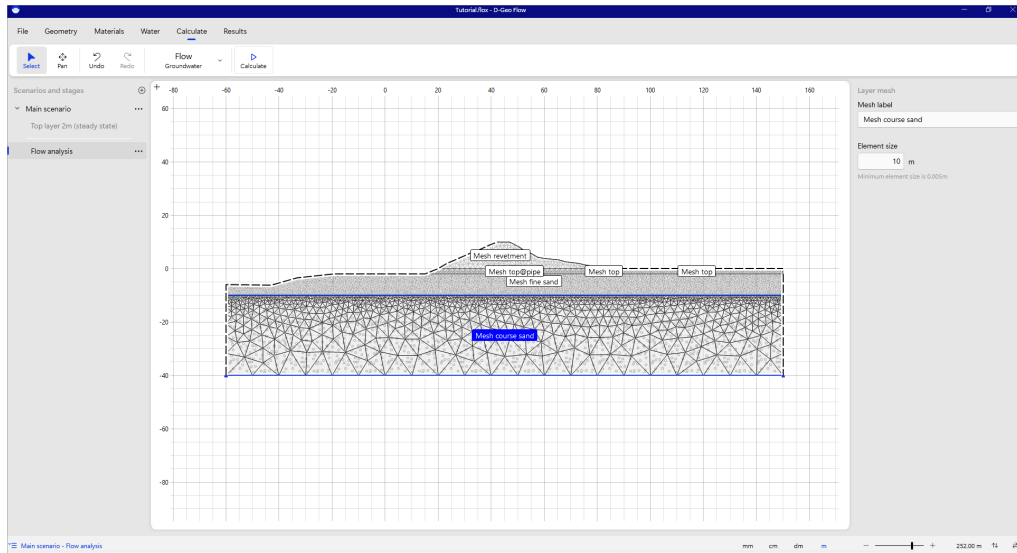


Figure 3.20 *Course mesh (good choice)*

Having a fine mesh, like in the figure below, increases the calculation time and gives some more precision and sometimes prettier results.

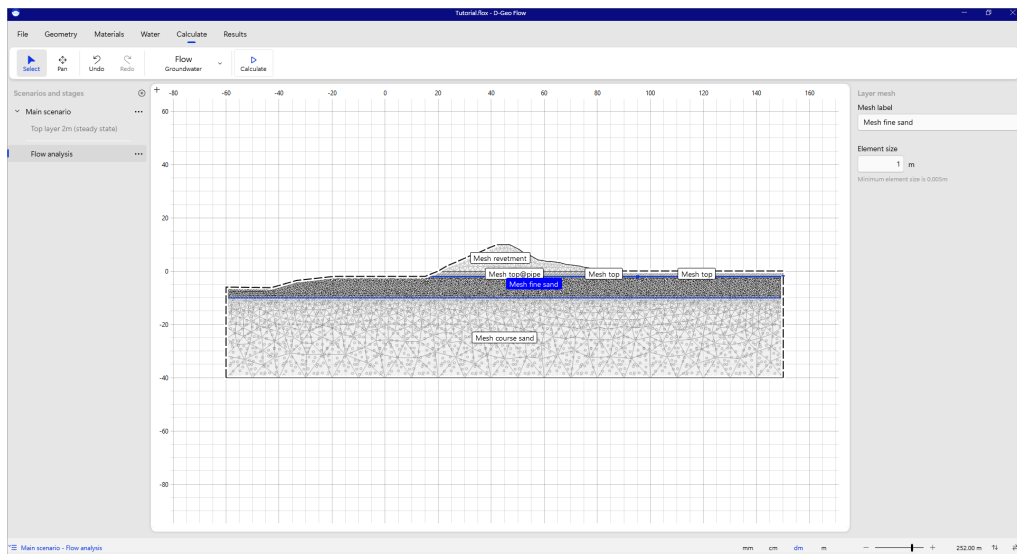


Figure 3.21 *Fine mesh for pretty results*

A calculation can be run by pressing the "Calculate" button in the ribbon.

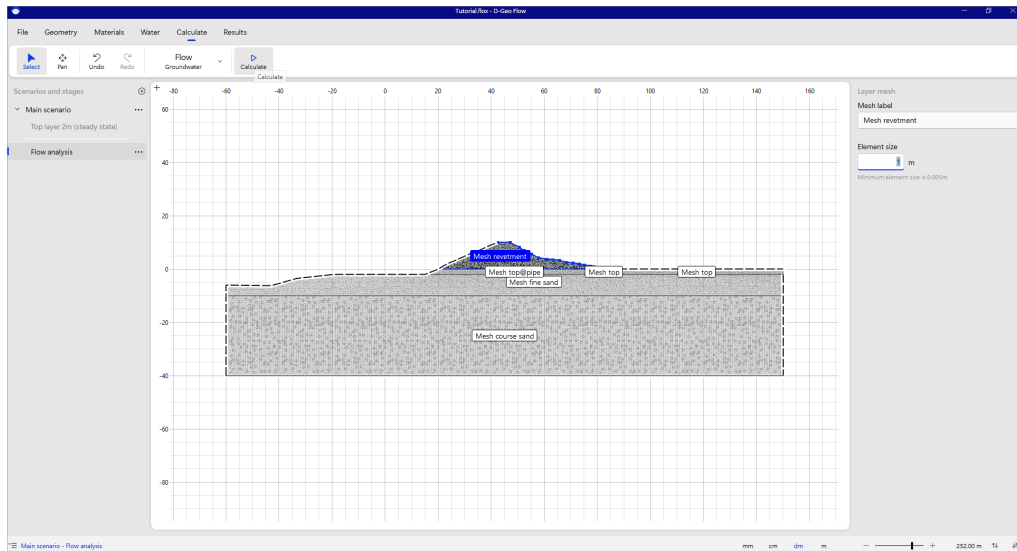


Figure 3.22 *Calculate*

After pressing this button, a pop up window shows that the analysis is running.

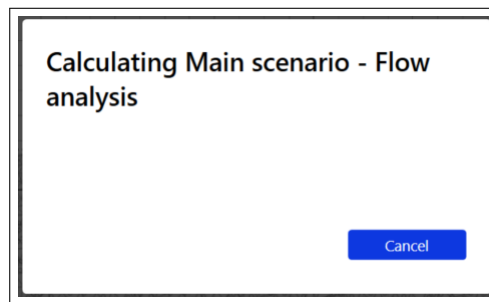


Figure 3.23 *The calculation is running*

After a successful run, the software goes to the "Results" tab and the overview of the results is shown. If the analysis does not produce results, the user will get feedback through the logging functionality why the run is not successful.

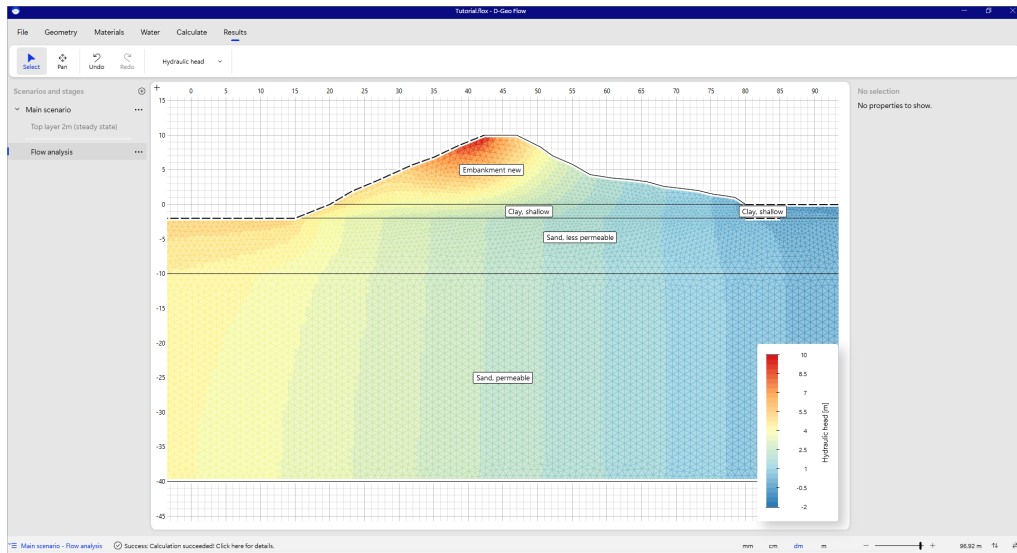


Figure 3.24 Results with fine mesh

By selecting an element, the user can see the results in the nodes and the average value. The user can inspect the hydraulic head, total pore pressures and total discharge

**Note:** The total discharge is the sum of the water flowing in and out of an element. This is zero for almost all elements. The only exception is near a boundary condition or near the pipe. There you will see there is an additional discharge.

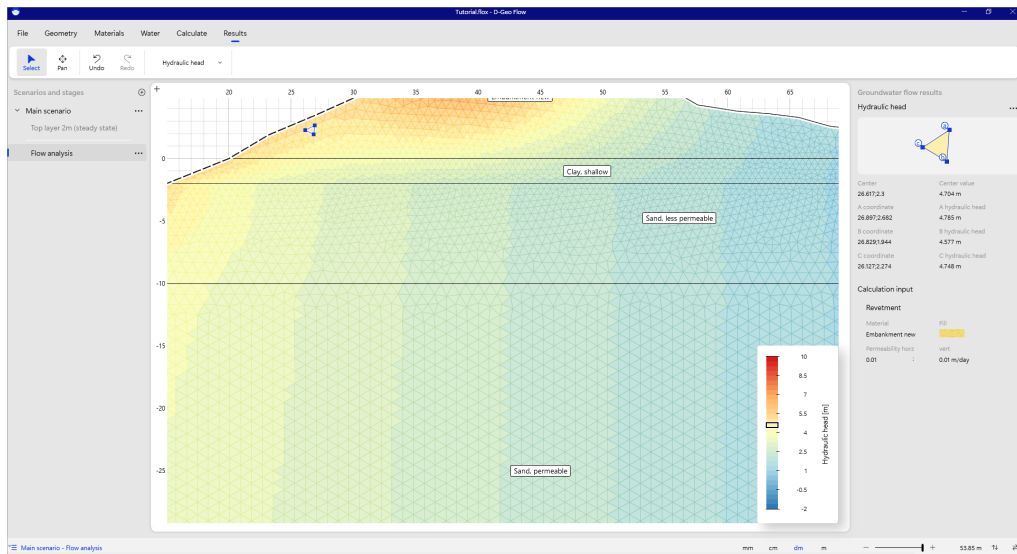


Figure 3.25 Inspect element of the results



### 3.4 Calculate the length of a pipe due to backward erosion pipping

Two new calculations will be defined, one to calculate the length of a pipe, given a head drop over the embankment, and one to calculate the critical head.

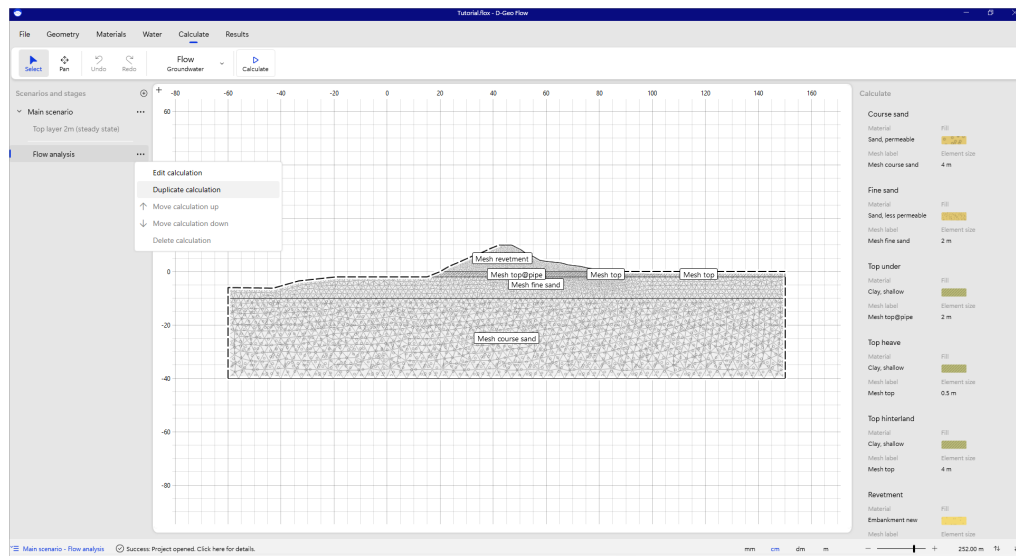


Figure 3.26 Duplicate calculation for pipe length and critical head

And a new scenario is defined to later on calculate the critical head without the top sand layer in the aquifer.

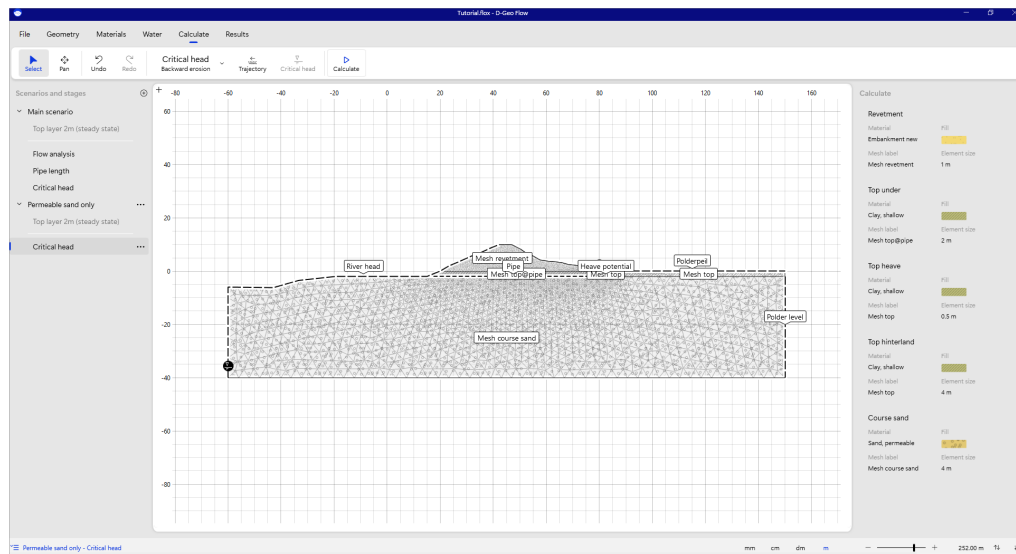


Figure 3.27 Duplicate scenario for other subsoil schematization

In the "Calculate" tab, we choose the "Pipe Length" analysis.

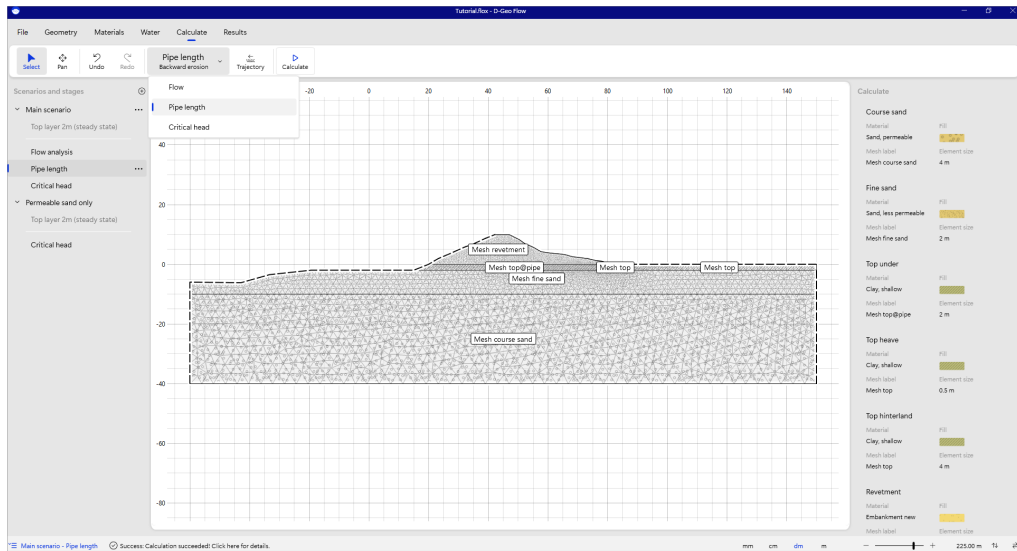


Figure 3.28 Select pipe length analysis

The pipe will run underneath the embankment.

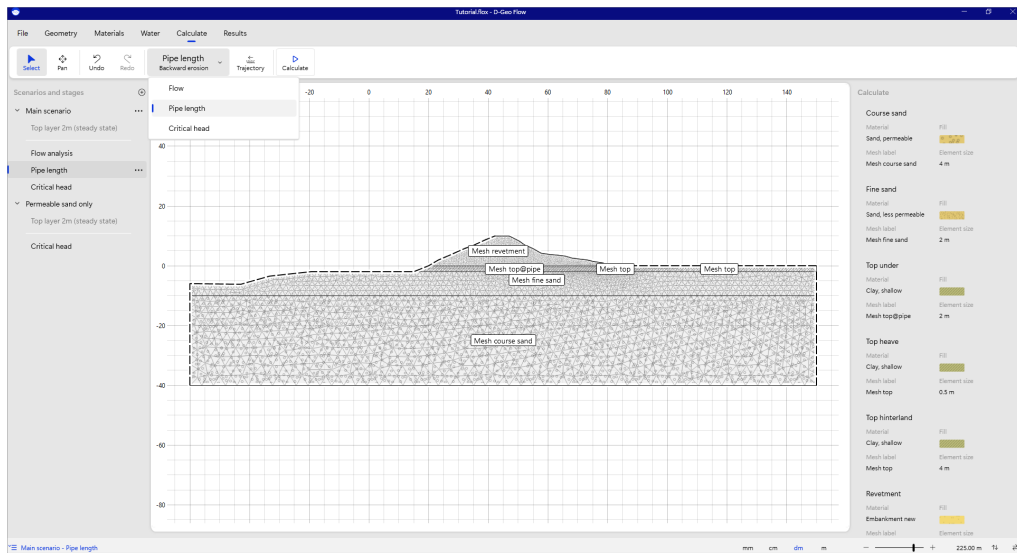


Figure 3.29 Define pipe trajectory

The trajectory of the pipe needs to be defined. The user can do so by clicking the "Trajectory" button, and click the begin and end point of the trajectory. The direction of the pipe needs to be correct. The user can change this in the inspector after selecting the pipe trajectory.

**Note:** Make sure the direction of the piping analysis is in line with the groundwater flow. No pipe will be found if the analysis is made in the wrong direction.



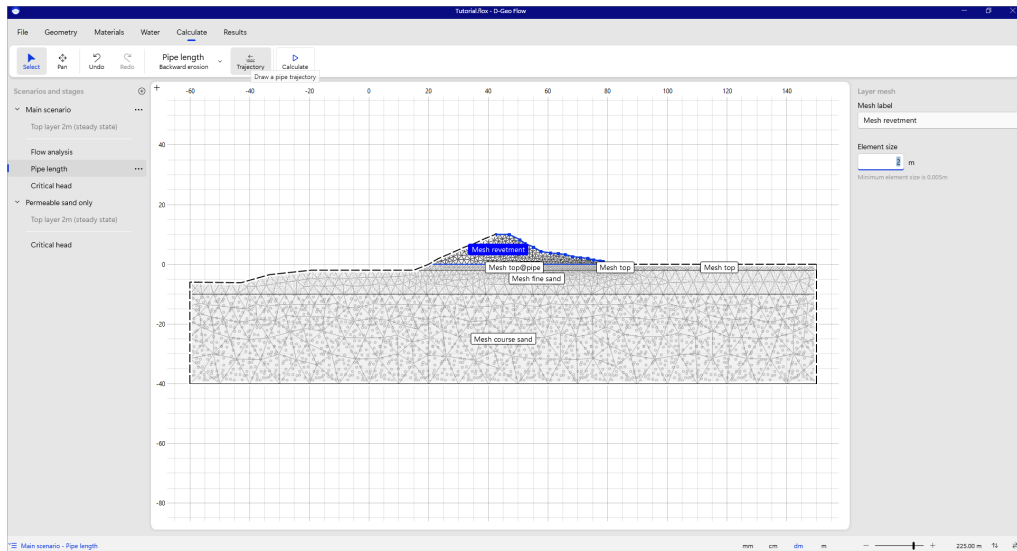


Figure 3.30 *Button on ribbon to define the pipe trajectory*

After defining the trajectory, the element size around the pipe can be set.

**Note:** The number of elements along the pipe is the most important indicator of the calculation time. Unfortunately, the result is mesh dependent. Usually, a mesh of about 1 meter along the pipe is sufficient. Starting with a coarser mesh can reduce the calculation time a lot. Investigating the effect of a mesh size of, for example, 0,5 meters is advised at least once during your project. This will increase the calculation time significantly.

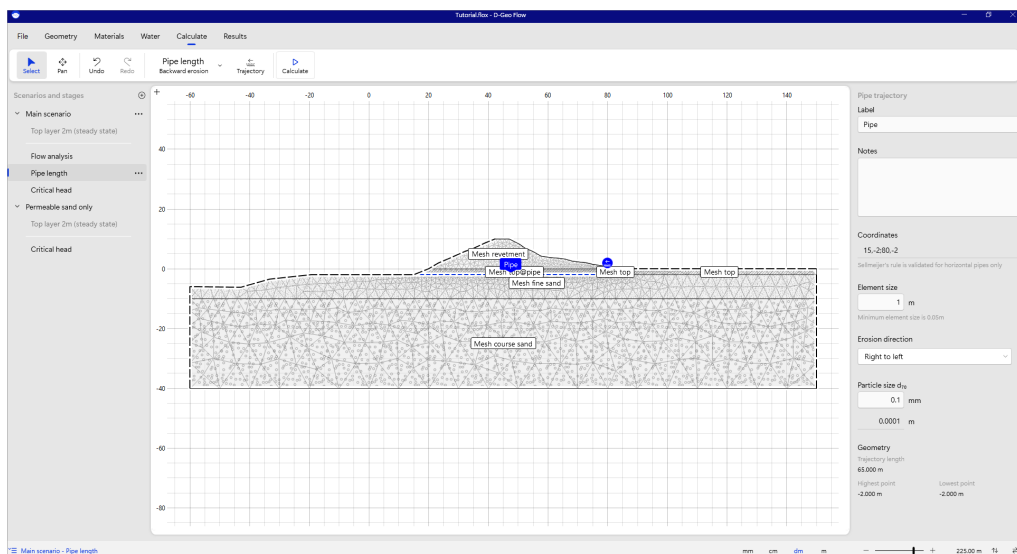


Figure 3.31 *Draw pipe trajectory*

After defining the trajectory of the pipe, the analysis can be run by pressing the "Calculate" button.

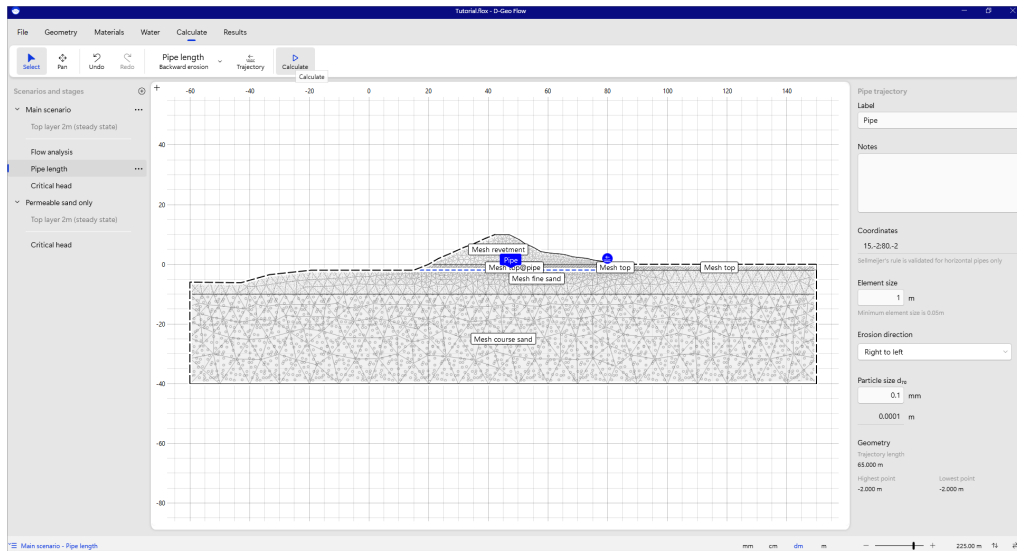


Figure 3.32 Run pipe length analysis

After a successful calculation, the results are shown. The length of the pipe is reported in the inspector and drawn on the canvas.

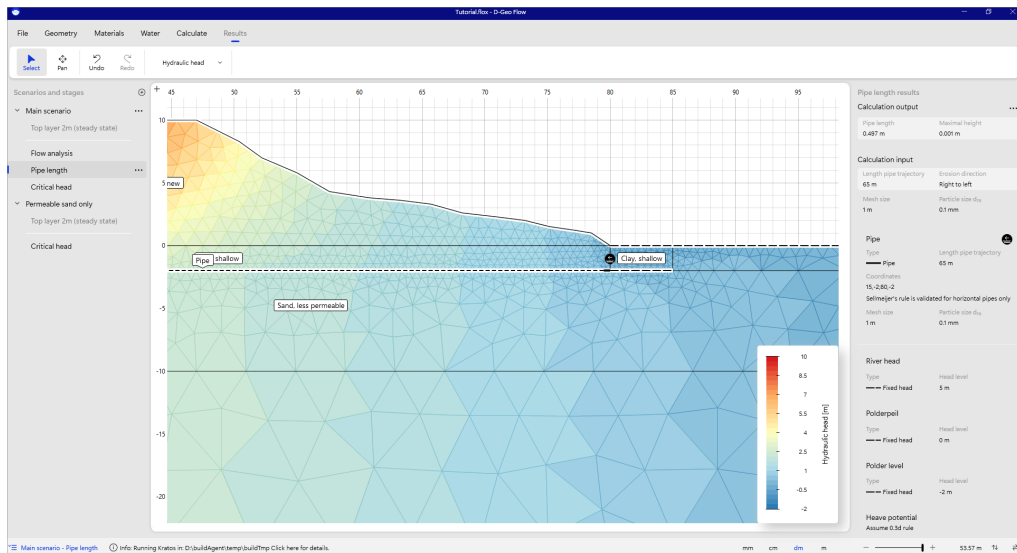


Figure 3.33 Inspect calculated pipe

Mesh elements can be inspected for more detailed information regarding the hydraulic head, total pore pressure and hydraulic discharge of the elements.

**Note:** The discharge gives relevant information right next to a pipe and next to boundary conditions. An element in the middle of the geometry, has no net discharge.



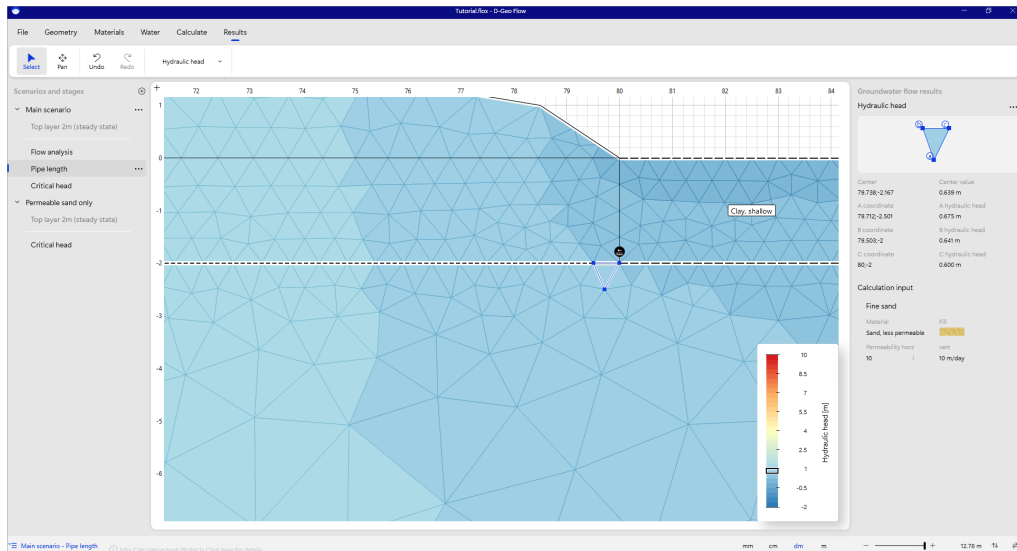


Figure 3.34 *Inspect element along pipe*

### 3.5 Determining the critical head

The "Critical Head" is the head level just before the head level where a pipe would reach the other side of the embankment. One step higher, and the pipe will continue to grow to the other side.

It can be calculated by selecting "Critical Head" in the "Calculate" tab. The user then needs to define which fixed head boundary condition to use when finding the critical head by increasing its head level. This is usually the river water level.

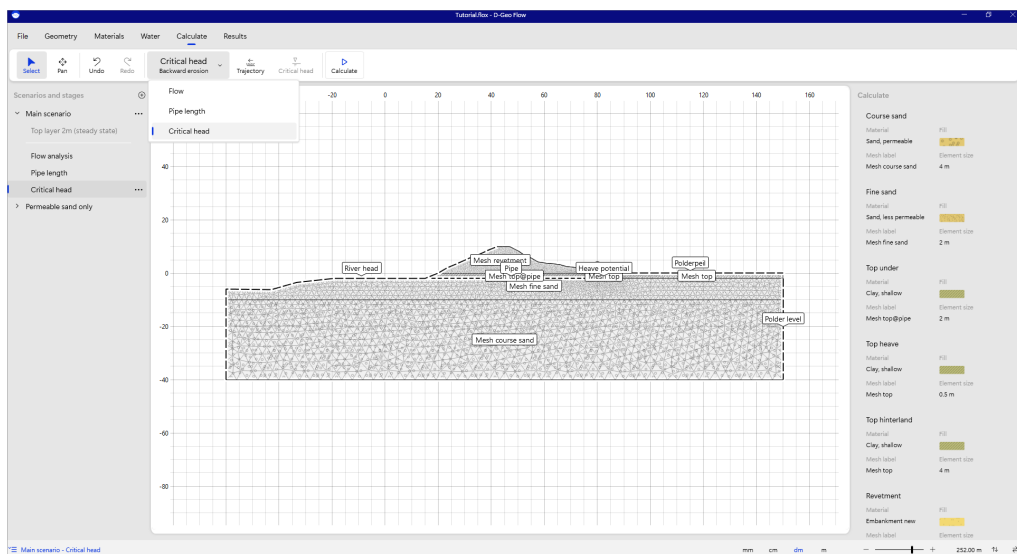


Figure 3.35 *Define critical head analysis*

After selecting a fixed head boundary, it can be marked as "Critical" in the ribbon. The user then needs to define the search area by entering a minimum head level, maximum head level and a step size.

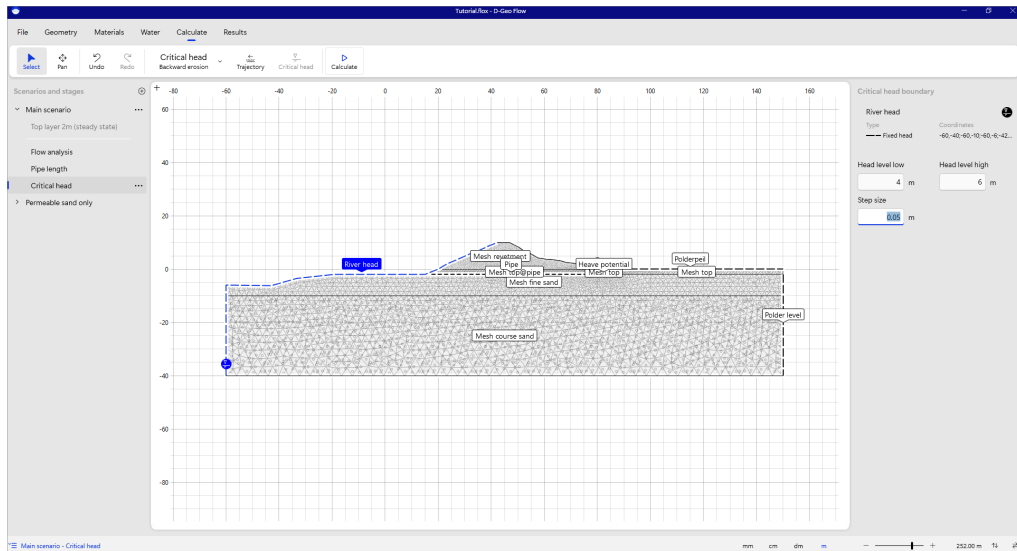


Figure 3.36 Define search scope and step size for critical head

**Note:** This search can take a long time. It is advised to search in larger steps first, and refine later on.



After calculating, the critical head is displayed on the canvas, along with the pipe length just before progressive failure.

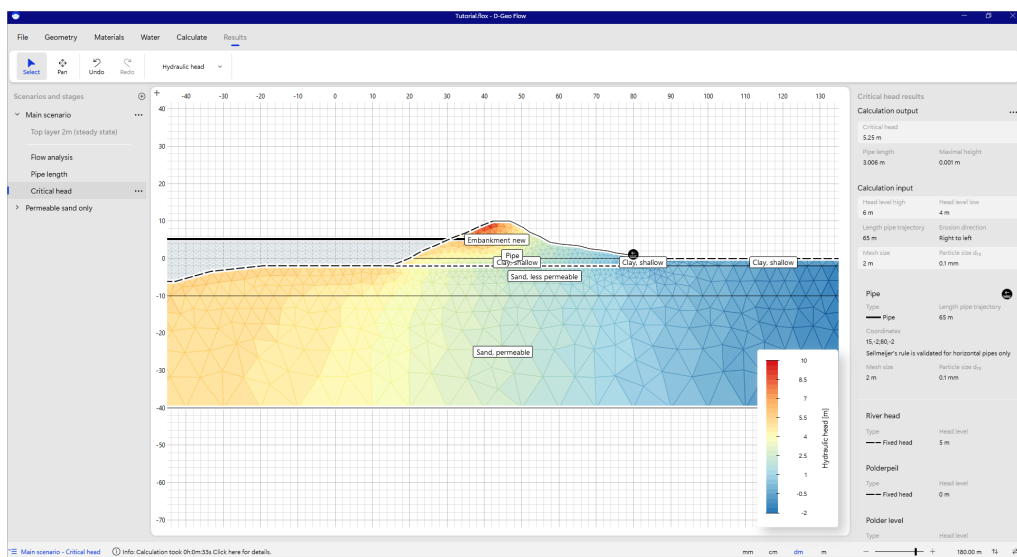


Figure 3.37 Result: critical head

Details can be inspected in the property inspector after selecting elements on the canvas.

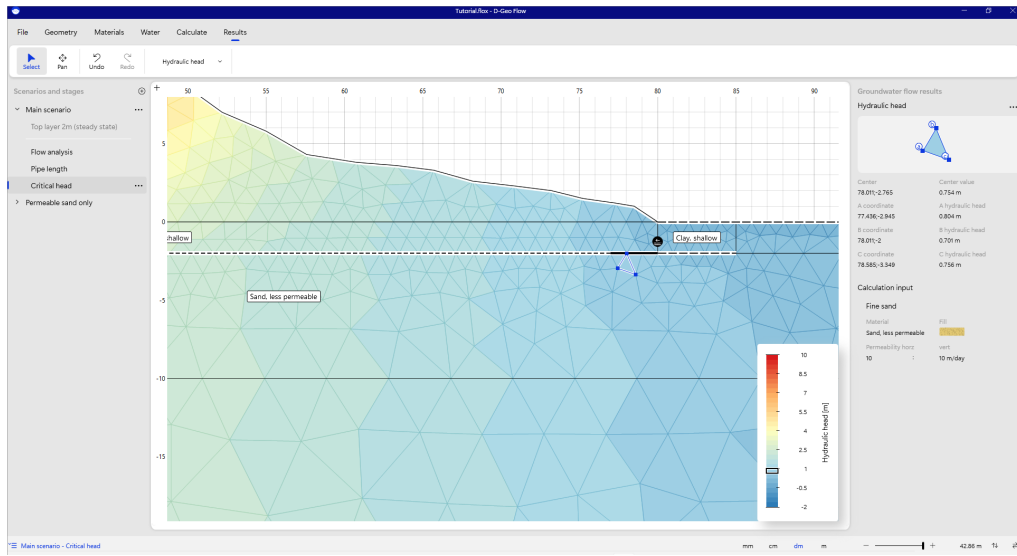


Figure 3.38 *Inspect element for critical head*



# Scientific Manual

## 4 Scientific background

### 4.1 FEM Piping background

The calculation process starts by checking the interface element located to the downstream side of the channel (element 1) next to the exit point. For a thickness of 0 mm the head gradient is due to the flow process through the subsurface. Increasing the thickness of the element increases the effective permeability of the element and decreases the gradient over the element. The height of the channel is increased stepwise until the condition becomes unstable. This unstable situation is associated with the first (right) point where the red line intersects the equilibrium line. At this point the element becomes active for the given head difference over the structure; a small increase in the channel height pushes the element in the unstable zone. The permeability of the element is increased until the element becomes stable again. The element thickness for which equilibrium holds is obtained by means of a bisection algorithm. This second (left) intersection point denotes a stable situation. If the height of the channel increases above this point, the element enters the stable zone.

The procedure continues by checking the element on the upstream side (element 2) of the first element (element 1). The height is increased sequentially until the line enters the unstable zone and the second intersection point is found, or a threshold value for the maximum channel height has been reached indicating that the element will not be activated. For the depicted situation, the applied head difference over the structure will not activate the second element. If equilibrium is not satisfied for the trial element however, then the height of the already activated elements is modified accordingly until equilibrium is satisfied within the entire pipe domain.

### 4.2 Translation of parameters for BOI usage

Table 4.1 Translation of parameters and defaults in D-Geo Flow

parameter	symbol	BOI name	Variabele in GUI	Value	unit
particle diameter	$D_{70}$	Naam NL	D-70	Variable	m
density of the grains	$\rho^g$	Naam NL	Volume gewicht korrels	Fixed	kg/m <sup>3</sup>
White's constant	$\eta$	Naam NL	Constante van White	Fixed	–
bedding angle	$\vartheta$	Naam NL	Rolhoek weerstand	Fixed	deg
intrinsic permeability	$\kappa$	Naam NL	Intrinsieke doorlatendheid	Fixed	
pore water density	$\rho^l$	Naam NL	Volume gewicht water	Fixed	kg/m <sup>3</sup>
dynamic viscosity water	$\mu^l$	Naam NL	Dynamische viscositeit water	Fixed	Ns/m <sup>2</sup>
gravitational acceleration	$g$	Naam NL	Zwaartekracht versnelling	Fixed	m/s <sup>2</sup>

Please note the units in the table above. In other BOI manuals, Kinematic viscosity is often measured as  $\mu/\rho^l$  in [m<sup>2</sup>/s]. Please enter the units in D-Geo Flow as requested in the user interface.

## 4.3 Default and recommended settings

Table 4.2 material parameter ranges

parameter	symbol	minimum	default	maximum	unit
particle diameter	$D_{70}$	$1.00 \cdot 10^{-4}$	$2.00 \cdot 10^{-4}$	$8.00 \cdot 10^{-4}$	m
density of the grains	$\rho^g$	$2.40 \cdot 10^3$	$2.65 \cdot 10^3$	$2.80 \cdot 10^3$	kg/m <sup>3</sup>
White's constant	$\eta$	0.20	0.25	0.30	–
bedding angle	$\vartheta$	30	37	50	deg
intrinsic permeability	$\kappa$	$10^{-16}$	$1.157 \cdot 10^{-11}$	$10^{-10}$	m <sup>2</sup>
pore water density	$\rho^l$	$10^3$	$10^3$	$10^3$	kg/m <sup>3</sup>
dynamic viscosity water	$\mu^l$	$8.0 \cdot 10^{-4}$	$1.3 \cdot 10^{-3}$	$2.0 \cdot 10^{-3}$	Ns/m <sup>2</sup>
gravitational acceleration	$g$	9.00	9.81	11.00	m/s <sup>2</sup>

## 4.4 Groundwater flow

### 4.4.1 Mathematical Model

The storage equation according to Richard's model follows from the mass balance equation, Darcy's law and empirical relations for water content and relative permeability as a function of pore pressure. This article adopts a Galerkin finite element discretization in space, in which the primary field variable is pore pressure  $p$  (N/m<sup>2</sup>)

Pore pressures are considered to be positive in compression, with index notation being used,  $x_i$  (m) denotes Cartesian coordinates in space,  $t$  (s) denotes time, and  $\Omega$  (m<sup>2</sup>) represents the flow domain with  $\Gamma$  (m) its boundary.

### 4.4.2 Subsurface Flow Equation

Flow through a partly saturated porous medium can be modeled considering conservation of mass and a generalization of Darcy's law, which can be written as

$$(\alpha + n\beta) S \frac{\partial p}{\partial t} + n \frac{dS}{dp} \frac{\partial p}{\partial t} + \frac{\partial q_i}{\partial x_i} = 0, \quad q_i = -\frac{k_r \kappa_{ij}}{\mu} \left( \frac{\partial p}{\partial x_j} - \rho^l g_j \right) \quad \text{on } \Omega^p, \quad (4.1)$$

where  $\alpha$  (m<sup>2</sup>/N) is the compressibility of the soil skeleton,  $\beta$  (m<sup>2</sup>/N) represents the compressibility of the pore water,  $n$  (–) denotes porosity and  $S$  (–) expresses the degree of saturation of the liquid phase in the void space. The compressibility for elastic behavior of the soil skeleton can be written as  $\alpha = 1/(\lambda + 2\nu)$ , where  $\lambda$  (N/m<sup>2</sup>) and  $\nu$  (N/m<sup>2</sup>) denote Lamé's constants. Specific discharge  $q_i$  (m/s) relates to relative permeability  $k_r$  (–), intrinsic permeability  $\kappa_{ij}$  (m<sup>2</sup>), dynamic viscosity of the liquid phase  $\mu$  (Ns/m<sup>2</sup>) and its density  $\rho^l$  (kg/m<sup>3</sup>). For a  $y$ -coordinate pointing in the opposite direction than the gravitational vector  $g_y$  is  $-9.81$  m/s<sup>2</sup>. Equation 4.1 is known as the storage equation. Two types of boundary conditions complete the problem definition; Dirichlet conditions prescribe the pressure on parts of the boundary and Von Neumann boundary conditions prescribe the derivative of the pressure or flux on the boundary. These first-type and second-type

boundary conditions read

$$p = \bar{p} \text{ on } \Gamma_1^p, \quad q_i n_i = -\bar{q} \text{ on } \Gamma_2^p, \quad q_i n_i = -s \text{ on } \Gamma_3^p, \quad (4.2)$$

where  $\Gamma_1^p$ ,  $\Gamma_2^p$  and  $\Gamma_3^p$  are disjoint parts of the boundary where the conditions apply,  $n_i$  (–) denotes the outward pointing normal to the boundary,  $\bar{p}$  (N/m<sup>2</sup>) is the prescribed pressure,  $\bar{q}$  (m/s) denotes the volumetric water flux over the boundary into the flow domain as a source term, and  $s$  (m/s) expresses a source term that will be used for coupling the subsurface flow equation to the flow equation for piping.

A weak form of equation 4.1 follows from multiplication by weighting functions  $N_a$  (–), which are attached to nodes of a finite element mesh and integration over the flow domain. The first weak form is given by

$$\int_{\Omega^p} N_a (\alpha + n\beta) S \frac{\partial \hat{p}}{\partial t} d\Omega + \int_{\Omega^p} N_a n \frac{dS}{dp} \frac{\partial \hat{p}}{\partial t} d\Omega = \int_{\Omega^p} N_a \frac{\partial}{\partial x_i} \left[ \frac{k_r \kappa_{ij}}{\mu} \left( \frac{\partial \hat{p}}{\partial x_j} - \rho^l g_j \right) \right] d\Omega, \quad (4.3)$$

where the caret above a symbol implies that the variable is an approximation. Application of Green's theorem to the term on the right hand side of equation 4.3 leads to

$$\begin{aligned} & \int_{\Omega^p} N_a (\alpha + n\beta) S \frac{\partial \hat{p}}{\partial t} d\Omega + \int_{\Omega^p} N_a n \frac{dS}{dp} \frac{\partial \hat{p}}{\partial t} d\Omega = \\ & - \int_{\Omega^p} \frac{k_r \kappa_{ij}}{\mu} \frac{\partial N_a}{\partial x_i} \left( \frac{\partial \hat{p}}{\partial x_j} - \rho^l g_j \right) d\Omega + \int_{\Gamma_{2 \cup 3}^p} N_a \frac{k_r \kappa_{ij}}{\mu} \left( \frac{\partial \hat{p}}{\partial x_j} - \rho^l g_j \right) n_i d\Gamma. \end{aligned} \quad (4.4)$$

Galerkin weighting and substitution of the interpolation functions gives

$$\begin{aligned} & \int_{\Omega^p} N_a N_b (\alpha + n\beta) S \frac{dp_b}{dt} d\Omega + \int_{\Omega^p} N_a N_b n \frac{dS}{dp} \frac{dp_b}{dt} d\Omega = - \int_{\Omega^p} \frac{k_r \kappa_{ij}}{\mu} \frac{\partial N_a}{\partial x_i} \frac{\partial N_b}{\partial x_j} p_b d\Omega \\ & + \int_{\Omega^p} \frac{k_r \kappa_{ij}}{\mu} \frac{\partial N_a}{\partial x_i} \rho^l g_j d\Omega + \int_{\Gamma_2^p} N_a N_b \bar{q}_b d\Gamma + \int_{\Gamma_3^p} N_a N_b s_b d\Gamma, \end{aligned} \quad (4.5)$$

after the Neumann boundary conditions were incorporated. Prescribed boundary fluxes are given at the nodal points. Implicit time integration yields

$$\begin{aligned} & \int_{\Omega^p} N_a N_b (\alpha + n\beta) S^{k+1} (p_b^{k+1} - p_b^k) d\Omega + \int_{\Omega^p} N_a N_b n \left( \frac{dS}{dp} \right)^{k+1} (p_b^{k+1} - p_b^k) d\Omega = \\ & - \Delta t \int_{\Omega^p} \frac{k_r^{k+1} \kappa_{ij}}{\mu} \frac{\partial N_a}{\partial x_i} \frac{\partial N_b}{\partial x_j} p_b^{k+1} d\Omega + \Delta t \int_{\Omega^p} \frac{k_r^{k+1} \kappa_{ij}}{\mu} \frac{\partial N_a}{\partial x_i} \rho^l g_j d\Omega \\ & + \Delta t \int_{\Gamma_2^p} N_a N_b \bar{q}_b^{k+1} + \Delta t \int_{\Gamma_3^p} N_a N_b s_b^{k+1} d\Gamma, \end{aligned} \quad (4.6)$$

where  $\Delta t$  is the time step size,  $k$  denotes the current time step and  $k + 1$  indicates the new step.

## 4.5 Pipe Length

#### 4.5.1 Flow Equation for Piping

Modeling of piping follows from the application of conservation of mass and considering Poiseuille flow through a horizontal erosion slit and can be written as

$$\frac{dq}{dx} + s = 0, \quad q = -\frac{a^3}{12\mu} \frac{dp}{dx} \quad \text{on } \Omega^c, \quad (4.7)$$

where  $a$  (m) denotes the height of the channel and  $s$  expresses a sink term. Here the extension of the pipe flow domain  $\Omega^c$  (m) corresponds to the boundary of the subsurface flow domain  $\Gamma^p$  (m) as equation 4.7 holds for a one-dimensional flow domain. Dirichlet conditions hold on the inflow and outflow point of the piping domain. Boundary conditions for the piping domain are given by

$$p = \bar{p} \quad \text{on } \Gamma_1^c. \quad (4.8)$$

The condition of limit equilibrium for the horizontally oriented erosion channel reads

$$a \frac{dp}{dx} = \frac{\pi}{3} (\rho^s - \rho^l) g d_{70} \eta \tan \vartheta. \quad (4.9)$$

The height of the erosion channel varies in between 0 and 3 grains in an experimental setup, 0 to 10 grains in a field test and 0 to 30 grains for an embankment.

Discretization of equation 4.7 follows the same steps as the discretization of equation 4.1, and the first weak form reads

$$\int_{\Omega^c} N_a \frac{d}{dx} \left( \frac{\kappa_p}{\mu} \frac{d\hat{p}}{dx} \right) d\Omega = \int_{\Omega^c} N_a s d\Omega, \quad (4.10)$$

where the equivalent intrinsic permeability  $\kappa_p$  (m<sup>3</sup>) has a scalar value that is given by  $\kappa_p = a^3/12$ . The second weak form is given by

$$\int_{\Omega^c} \frac{\kappa_p}{\mu} \frac{dN_a}{dx} \frac{d\hat{p}}{dx} d\Omega = - \int_{\Omega^c} N_a s d\Omega, \quad (4.11)$$

and the final form follows from the substitution of interpolation functions

$$\int_{\Omega^c} \frac{\kappa_p^{k+1}}{\mu} \frac{dN_a}{dx} \frac{dN_b}{dx} p_b^{k+1} d\Omega = - \int_{\Omega^c} N_a N_b s_b^{k+1} d\Omega. \quad (4.12)$$

The condition of limit equilibrium for a horizontally oriented erosion channel is evaluated at the integration points and is given by

$$\kappa_p^{k+1} = \frac{1}{12} (a^{k+1})^3, \quad a^{k+1} \frac{dN_b}{dx} p_b^{k+1} = \frac{\pi}{3} (\rho^s - \rho^l) g d_{70} \eta \tan \vartheta. \quad (4.13)$$

#### 4.5.2 Coupled Flow Equation

The sink term in equation 4.12, where  $s$  is positive for flow out of the channel, provides a source term in equation 4.6, where  $s$  is positive for flow into the subsurface, and vice versa. Linear one-dimensional elements are used for solving equation 4.12, whereas

linear triangular elements discretize the flow domain attached to equation 4.6. Combining both equations yields

$$\begin{aligned} & \int_{\Omega^p} N_a N_b (\alpha + n\beta) S^{k+1} (p_b^{k+1} - p_b^k) d\Omega + \int_{\Omega^p} N_a N_b n \left( \frac{dS}{dp} \right)^{k+1} (p_b^{k+1} - p_b^k) d\Omega = \\ & - \Delta t \int_{\Omega^p} \frac{\kappa_r^{k+1} \kappa_{ij}}{\mu} \frac{\partial N_a}{\partial x_i} \frac{\partial N_b}{\partial x_j} p_b^{k+1} d\Omega + \Delta t \int_{\Omega^p} \frac{\kappa_r^{k+1} \kappa_{ij}}{\mu} \frac{\partial N_a}{\partial x_i} \rho^l g_j d\Omega \\ & - \Delta t \int_{\Gamma_3^p} \frac{\kappa_p^{k+1}}{\mu} \frac{\partial N_a}{\partial x_i} \frac{\partial N_b}{\partial x_j} p_b^{k+1} d\Omega + \Delta t \int_{\Gamma_2^p} N_a N_b \bar{q}_b^{k+1} . \quad (4.14) \end{aligned}$$

The interface elements align along the horizontal axis. Picard iteration over  $\kappa_r^{k+1}$  and  $S^{k+1}$  are used to resolve the non-linearities in the system of equations. Non-linearities in  $\kappa_p^{k+1}$  are resolved sequentially. For a given trial channel height, equation 4.13 provides the equivalent intrinsic permeability, which is attached to the relevant interface element. The product of channel height and pressure gradient are then compared with the equilibrium condition given by the second part of equation 4.13.

## 4.6 Critical Head

The critical head denotes the river level at which a pipe breaks through. It is calculated by increasing a critical head boundary until the pipe reaches the other side. The last iteration before breakthrough is reported in the User Interface.

It is advised first to find the critical head with a rather course mesh and bigger iteration steps. beware that the pipe length and thereby the critical head is mesh dependent, once the critical head is almost known, one can decrease the mesh coarseness and the step size.

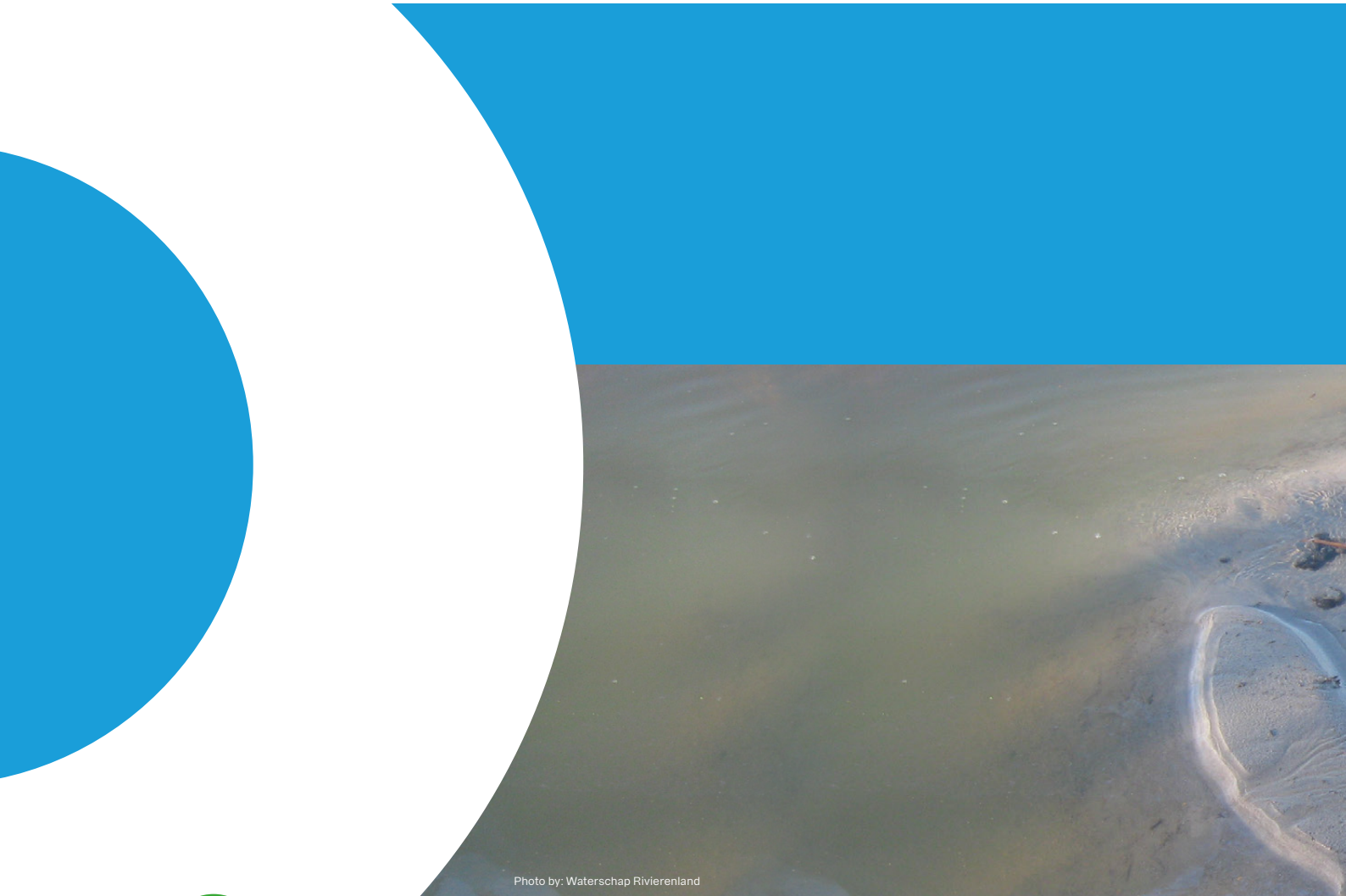


Photo by: Waterschap Rivierenland



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